AP Calculus BC Prerequisite Knowledge

Please review these ideas over the summer as they come up during our class and we will not be reviewing them during class. Also, I feel free to quiz you at any time on these facts and procedures.

When you join me in the fall, I will using the TI Nspire CX **CAS** calculator as our demonstration calculator. While you are welcome to use any graphing calculator, I won't be lecturing about the other calculators in class. You may purchase the TI Nspire CX **CAS** on Amazon.com for approximately $140.

If can wait until the fall, you can order one from me for $130. There is an order form on the back of this page.

As you look through these pages, feel free to ask me, other math teachers and your fellow future calculus students for clarifications.

Email: todd1@toddfadoir.com.
Cell: 313-799-2252
Website: www.toddfadoir.com/apcalc

We'll be using *Calculus: Graphical, Numerical, Algebraic.* Third Edition by Finney, Demana, Waits, & Kennedy.

Have a great summer!

Sincerely, Mr. Fadoir

Order for TI Nspire CX CAS, $130

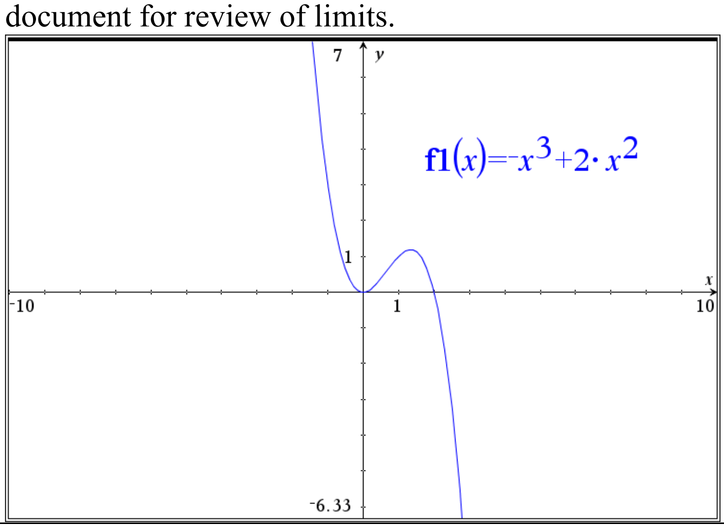
Cash or Check (payable to Berkley High School)

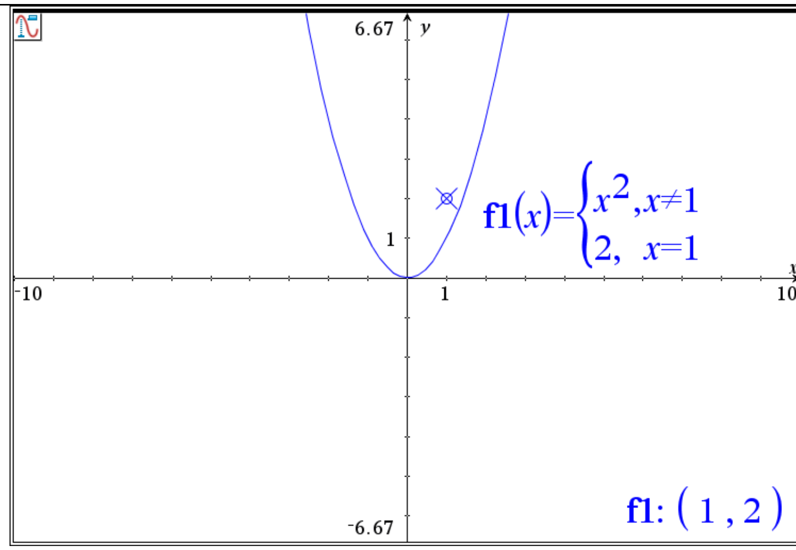
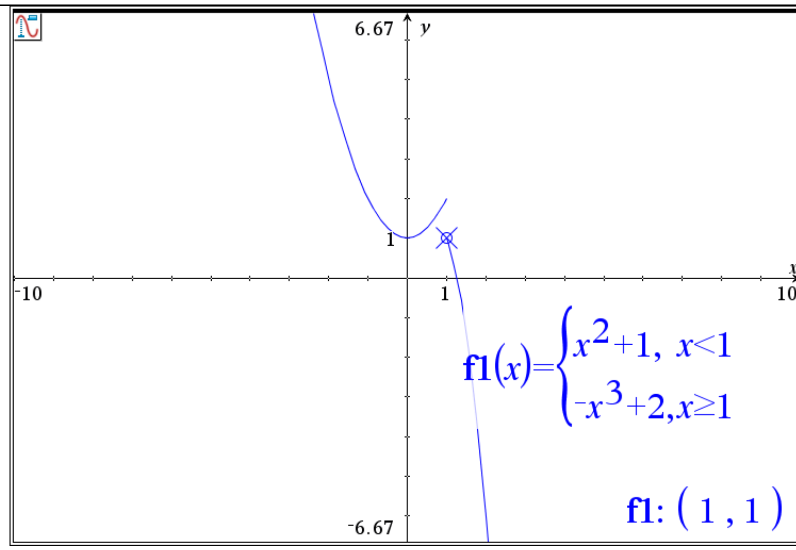
Student's Name: PRINT ____

Cash _______ Check # ______

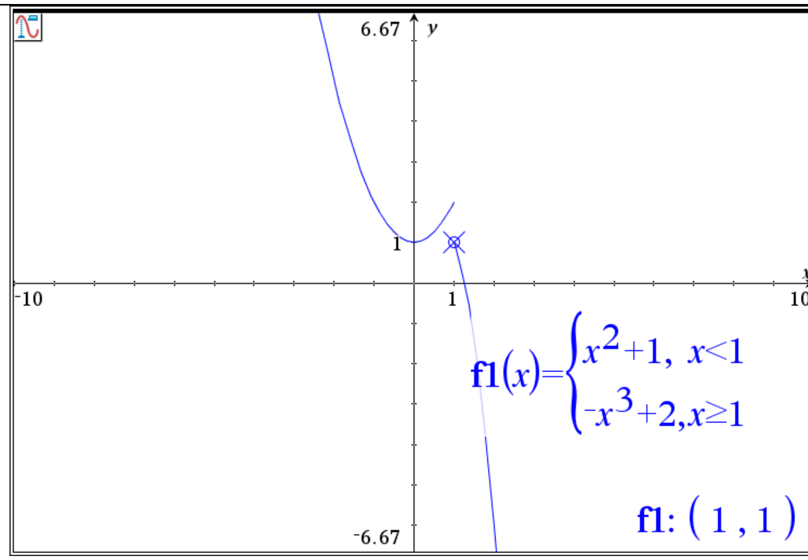
Name of check if different than student's: ____

Return to Mr. Fadoir by September 19, 2014

Algebra	
Find the equation of a line given a slope (m) and a point (x_0, y_0)	I know your tendency to want to create the equation in the form $y = mx + b$, I would like you to start using the form $y = y_0 + m(x - x_0)$.
What is the equation of a line with slope $m = 5$ that goes through the point $(3,4)$	$y = 4 + 5(x - 3)$. Done. Leave it in this form.
What is the equation of a line that goes through the points (x_0, y_0) and (x_1, y_1).	$m = \frac{y_1 - y_0}{x_1 - x_0}$, then use m in the equation $y = y_0 + m(x - x_0)$.
What is the equation of a line that goes through the points $(3,4)$ and $(5,8)$.	$m = \frac{8-4}{5-3} = \frac{4}{2} = 2$, then use 2 in the equation $y = 8 + 2(x - 5)$. Also $y = 4 + 2(x - 3)$ works.
When graphing polynomials, factor to find the zeroes. If the polynomials have factors with odd powers, the polynomial changes sign at the corresponding roots. If the polynomials have factors with even powers, the polynomial doesn't change sign at the corresponding root.	Ex. $y = -x^3 + 2x^2$ Factor to find the zeroes $y = -x^2(x - 2)$ There are at $x = 0$ and $x = 2$. Also, the factor that provide the zeroes at $x = 0$ is to an even power, so sign change at $x = 0$. The factor that provides the zero at $x = 2$ is to an odd power, so there is a sign change at $x = 2$ Lastly, $\lim_{x \to -\infty} (-x^3 + 2x^2) = \infty$, so the function starts positive, touches axis at $x = 0$, then goes through the axis at $x = 2$. Let's confirm by graphing. See later in the document for review of limits. 

Limits	
When we ask $\lim_{x\to a} f(x)$, we are asking, what value does $f(x)$ take on when we infinitesimally close to *a*, as compared to the question, what is the value of $f(a)$.	 The value $f_1(1)=2$, even though the $\lim_{x\to 1} f_1(x)=1$
Limits can be “one-sided”. $\lim_{x\to a^-} f(x)$ asks the question, “what value does *f* give as we get infinitesimally close to *a* from the left hand (or negative side). $\lim_{x\to a^+} f(x)$ asks the question, “what value does *f* give as we get infinitesimally close to *a* from the right hand (or positive side). When the left-hand limit and the right-hand do not agree, we say the overall limit does not exist.	 $\lim_{x\to 1^-} f_1(x)=2$, $\lim_{x\to 1^+} f_1(x)=1$, $\lim_{x\to 1} f_1(x)$ does not exist

<table>
<tr><th colspan="2">Limits</th></tr>
<tr><td>We say the function f is continuous at a if three conditions are met.

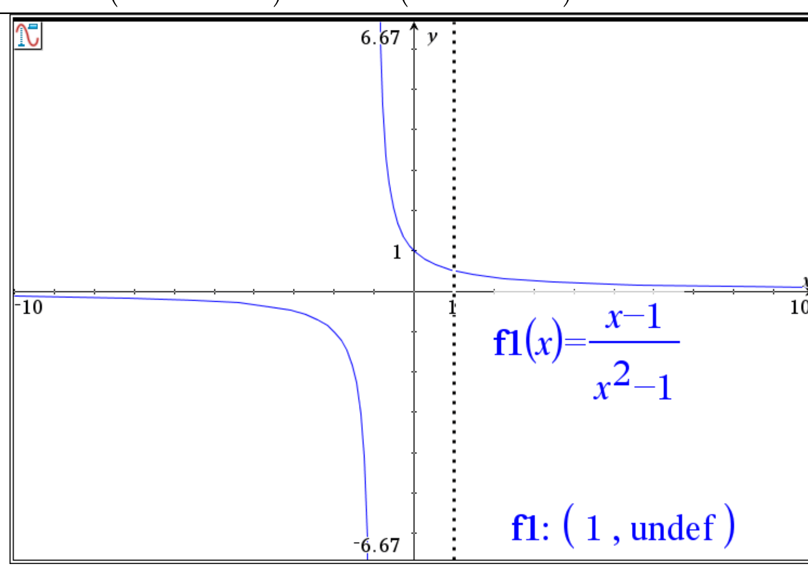
1. $f(a)$ exists.
2. $\lim_{x\to a} f(x)$ exists.
3. $\lim_{x\to a} f(x) = f(a)$
</td><td>


f_1 is not continuous at $x=1$ because $\lim_{x\to a} f_1(x)$ does not exist.
</td></tr>
<tr><td>For continuous functions, the value of the function is the value of the limit.</td><td>$\lim_{x\to 2} x^2 = 4$</td></tr>
<tr><td>For some discontinuous functions, the limit's value can be found through algebraic manipulation.</td><td>
Example 1:

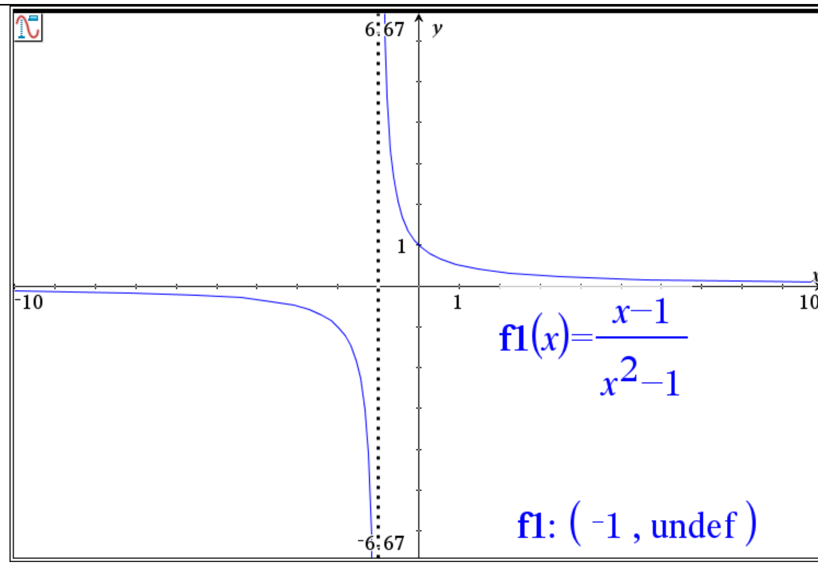
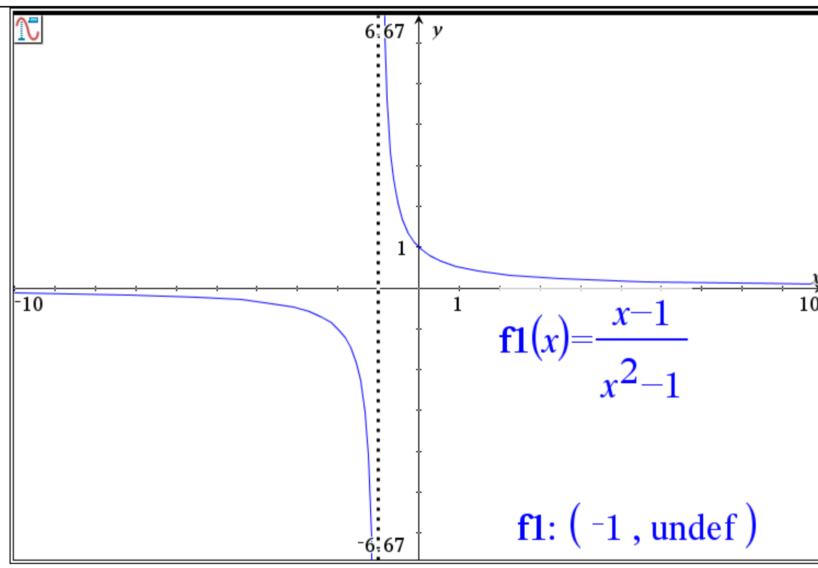
$$\lim_{x\to 1}\frac{x-1}{x^2-1} = \lim_{x\to 1}\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} = \lim_{x\to 1}\frac{1}{(x+1)} = \frac{1}{2}$$

Example 2:

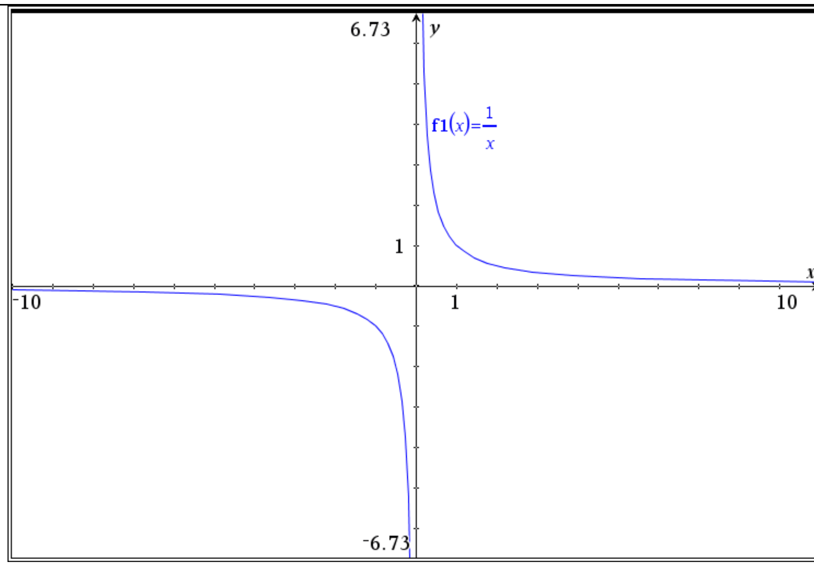
$$\lim_{x\to 0}\frac{\sqrt{5+x}-\sqrt{5}}{x} = \lim_{x\to 0}\frac{\sqrt{5+x}-\sqrt{5}}{x}\cdot\frac{\sqrt{5+x}+\sqrt{5}}{\sqrt{5+x}+\sqrt{5}}$$

$$= \lim_{x\to 0}\frac{5+x-5}{x\left(\sqrt{5+x}+\sqrt{5}\right)} = \lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}\left(\sqrt{5+x}+\sqrt{5}\right)} = \frac{1}{2\sqrt{5}}$$
</td></tr>
<tr><td>When the limit can be found through algebraic manipulation, the graph of these functions appear with removable discontinuities.</td><td>


$$\lim_{x\to 1}\frac{x-1}{x^2-1} = \frac{1}{2}$$
</td></tr>
</table>

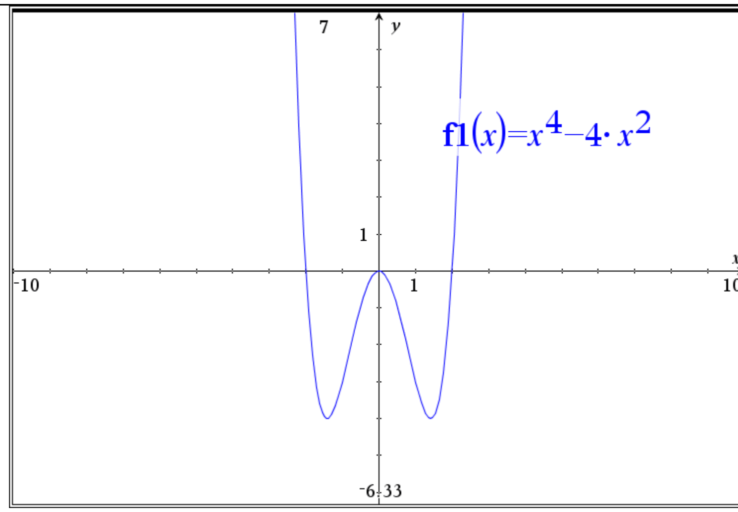
Limits	
Some limits can't be found using the algebraic manipulation technique.	 $\lim_{x\to -1}\frac{x-1}{x^2-1}$ does not exist.
The Intermediate Value Theorem states that if a function is continuous on the interval $[a,b]$ and you choose any value between $f(a)$ and $f(b)$ (let's call it L), then there exists a c in the interval (a,b) such that $f(c)=L$.	Example: Show that $f(x)=x^3$ has at least one zero. Condition check: First we acknowledge that f is continuous on $[-1,1]$ and that $f(-1)=-1$ and $f(1)=1$. 0 is a value between -1 and 1. Consequence: Therefore f must take on the value 0 somewhere on the interval $(-1,1)$.
Some limits have unbounded behavior in a particular direction. While we say their limits do not exist, we also say their limit approach infinity. When a function has a limit that approaches infinity as x approaches a finite value, we say the function has a vertical asymptote.	 $\lim_{x\to -1^+}\frac{x-1}{x^2-1}$ does not exist and $\lim_{x\to -1^+}\frac{x-1}{x^2-1}=\infty$ $\lim_{x\to -1^-}\frac{x-1}{x^2-1}$ does not exist and $\lim_{x\to -1^-}\frac{x-1}{x^2-1}=-\infty$ f_1 has a vertical asymptote at $x=-1$

Limits

We can ask "what happens to a function f when x gets arbitrarily large", and we use the notation $\lim_{x\to\infty} f(x)$. If the function gets closer to a particular value, we say the function has a horizontal asymptote.	 $\lim_{x\to\infty} f_1(x)=0$. $y=0$ is a horizontal asymptote..
Many times, we take the limit of a rational functions; rational functions are functions in the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials. When the order of $P(x)$ is larger than $Q(x)$, $\lim_{x\to\infty}\frac{P(x)}{Q(x)}=\infty$. When the order of $P(x)$ is smaller than $Q(x)$, $\lim_{x\to\infty}\frac{P(x)}{Q(x)}=0$. When the order of $P(x)$ is equal than $Q(x)$, $\lim_{x\to\infty}\frac{P(x)}{Q(x)}$ is found by looking at the ratio of the coefficients of the highest terms.	Example 1: $\lim_{x\to\infty}\frac{x^3-1}{2-3x^2}=\infty$ Example 2: $\lim_{x\to\infty}\frac{3x^2-1}{1-x^3}=0$ Example 3: $\lim_{x\to\infty}\frac{3x^3-1}{1-x^3}=\frac{3}{-1}=-3$

Functions

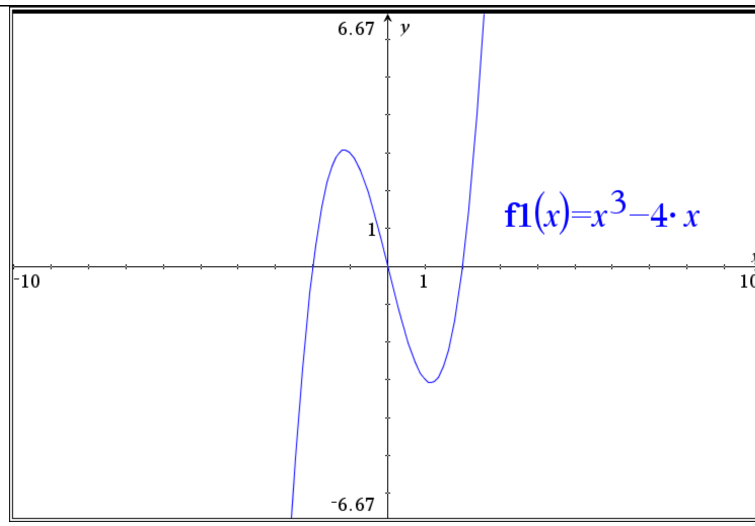
Some function follow the rule that $f(a)=f(-a)$. We call these functions "even" functions. They have symmetry about the *y*-axis.



$f_1(1)$ and $f_1(-1)$ both equal -3.

$f_1(2)$ and $f_1(-2)$ both equal 0.

Some function follow the rule that $f(a)=-f(-a)$. We call these functions "odd" functions. They have symmetry about the origin. Symmetry about the origin means that the midpoint between two corresponding points is the origin.



$f_1(1)=-3$ and $f_1(-1)=3=-f_1(-1)$.

$f_1(3)=15$ and $f_1(-3)=-15=-f_1(3)$.

If $f\left(f^{-1}(x)\right)=x$ and $f^{-1}\left(f(x)\right)=x$, we say the f and f^{-1} are inverse functions. We use inverse functions to "undo" the other function when solving equations. e^x and $\ln(x)$ are inverse functions. x^2 and $\sqrt{x}$ are inverse functions. $\sin x$ and $\sin^{-1} x$ (sometimes called $\arcsin x$) are inverse functions.

Example 1:

$e^x=10$

$\ln\left(e^x\right)=\ln(10)$

$x=\ln(10)$

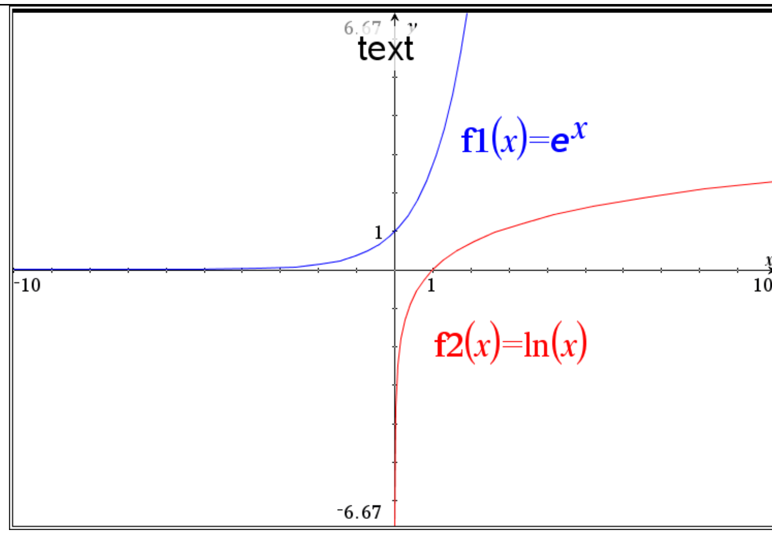
Example 2:

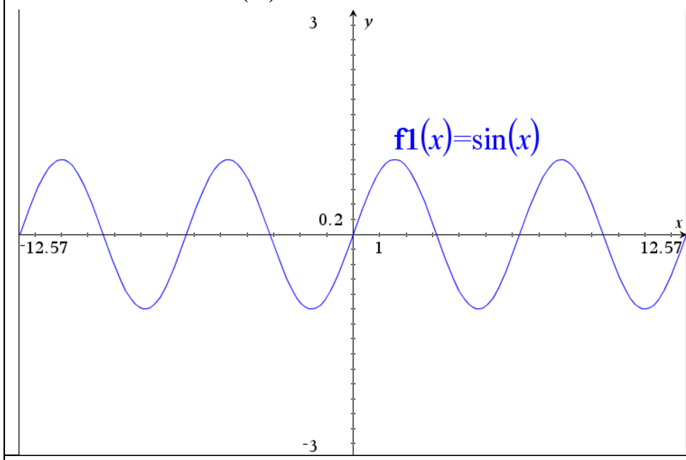
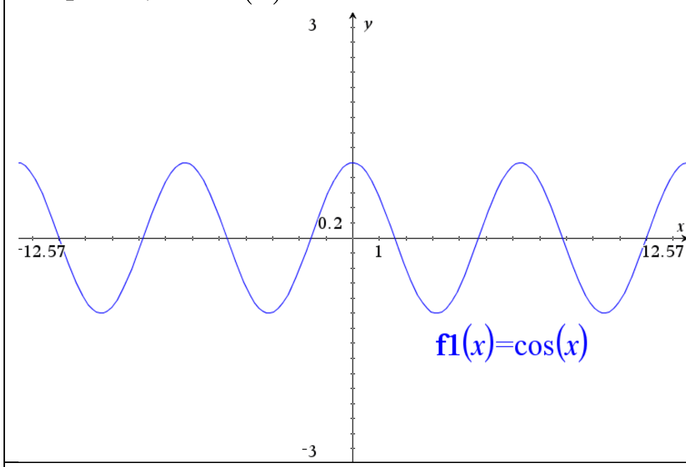
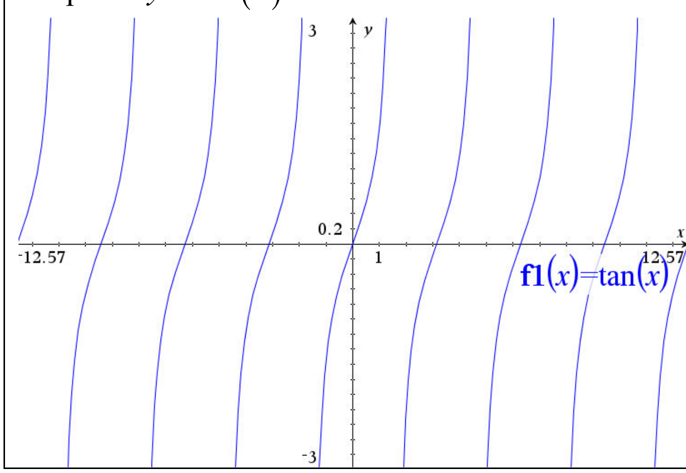
$\ln x=10$

$e^{\ln x}=e^{10}$

$x=e^{10}$

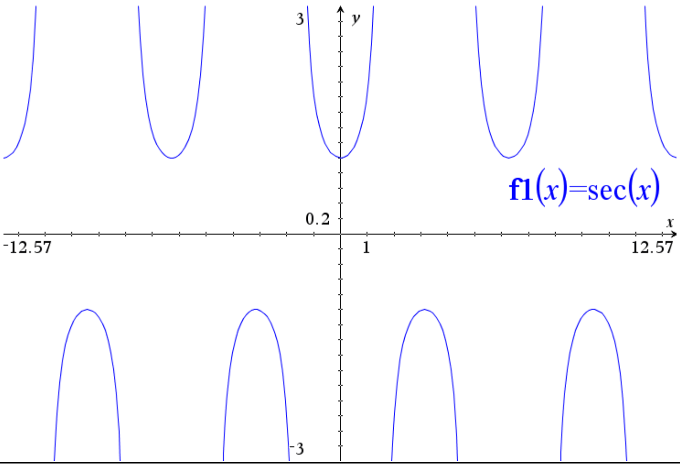
We used the fact that ln and e^x are inverse functions to isolate *x*.

Functions	
For every point on function *f*, the is a corresponding point on f^{-1} , the inverse function of f . Graphically, the function f is the reflection of the inverse function f^{-1} over the line $y = x$.	 $f_1(x) = e^x$, $f_2(x) = \ln(x)$. Because the point $(1,e)$ is on the graph of f_1, then the point $(e,1)$ is on the graph of f_2 **.** Because the point $(1,0)$ is on the graph of f_2 , the point $(0,1)$.
Rules for logs $\ln(a) + \ln(b) = \ln(a \cdot b)$ $\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$ $\ln(a^b) = b \cdot \ln(a)$ $\log_b a = \frac{\ln a}{\ln b}$ $\ln e = 1$ $\ln 1 = 0$	Example 1: $\ln(3e) = \ln(3) \cdot \ln(e) = \ln(3)$ Example 2: $\ln(x-1) - \ln(x+2) = \ln\left(\frac{x-1}{x+2}\right)$ $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+2) = \ln(x+1)^{\frac{1}{2}} + \ln(x+2)^{\frac{1}{2}}$ Example 3: $= \ln\left(\frac{(x-1)^{\frac{1}{2}}}{(x+2)^{\frac{1}{2}}}\right) = \ln\left(\sqrt{\frac{(x-1)}{(x+2)}}\right)$

Trig (Note: we work exclusively in radians in AP Calculus)	
Graph of $y = \sin(x)$  	Notes: • The function has zeroes at $x = n\pi, n$ is an integer. • The function has a maximum value of 1. • The function has a minimum value of -1. • The function is odd.
Graph of $y = \cos(x)$  	Notes: • The function has zeroes at $x = n\pi + \frac{\pi}{2}, n$ is an integer. • The function has a maximum value of 1. • The function has a minimum value of -1. • The function is even.
Graph of $y = \tan(x)$  	Notes: • $\tan(x) = \frac{\sin(x)}{\cos(x)}$ • The function has zeroes wherever $\sin(x) = 0$. • The function is undefined when $\cos(x) = 0$. • The function is increasing. • The function is odd.

Trig (Note: we work exclusively in radians in AP Calculus)

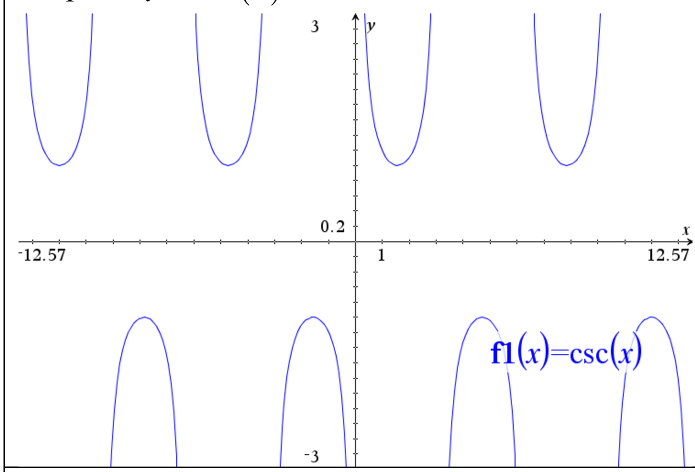
Graph of $y = \sec(x)$



Notes:

- $\sec(x) = \dfrac{1}{\cos(x)}$
- The function has local maximum of -1.
- The function has local minimum of 1.
- The function is undefined wherever $\cos(x) = 0$.
- The function is even, just like $\cos(x)$.

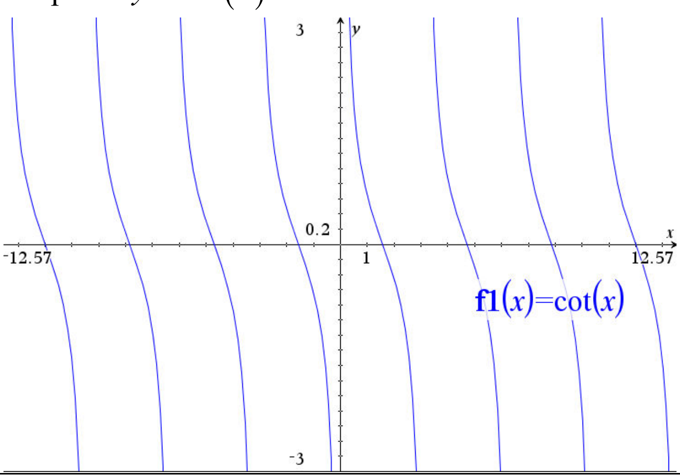
Graph of $y = \csc(x)$



Notes:

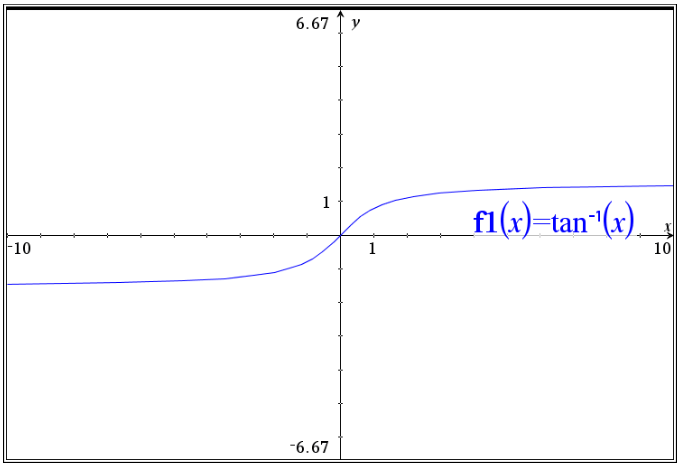
- $\csc(x) = \dfrac{1}{\sin(x)}$
- The function has local maximum of -1.
- The function has local minimum of 1.
- The function is undefined wherever $\cos(x) = 0$.
- The function is odd, just like $\sin(x)$.

Graph of $y = \cot(x)$



Notes:

- $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$
- The function is undefined when $\sin(x) = 0$.
- The function is always decreasing.
- The function is odd, just like $\sin(x)$.

Trig (Note: we work exclusively in radians in AP Calculus)	
Graph of $y = \tan^{-1}(x)$  	Notes: • In the normal tangent function, you give an angle, and you get back a ratio. For the inverse tangent function, you give a ratio and get back an angle. • $\tan^{-1}(1) = \frac{\pi}{4}$ because $\tan\left(\frac{\pi}{4}\right) = 1$. • $y = \frac{\pi}{2}$ is a horizontal asymptote because $\lim_{x\to\infty} \tan^{-1}(x) = \frac{\pi}{2}$. • $y = -\frac{\pi}{2}$ is a horizontal asymptote because $\lim_{x\to-\infty} \tan^{-1}(x) = -\frac{\pi}{2}$. • The function is odd.
You should be able to state the value of any of the big six trig functions for any multiple of $\frac{\pi}{6}$ from -2π to 2π and any multiple of $\frac{\pi}{4}$ from -2π to 2π. These are the values you memorized on the unit circle.	$\cos(0) = 1$; $\sin(0) = 0$; $\tan(0) = 0$; $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$; $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$; $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$; $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$; $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$; $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$; $\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$; $\tan\left(\frac{\pi}{4}\right) = 1$; $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$; $\cos\left(\frac{\pi}{2}\right) = 0$; $\sin\left(\frac{\pi}{2}\right) = 1$; $\tan\left(\frac{\pi}{2}\right)$ is und.; $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$; $\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$; $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$; and more…
Trig Pythagorean Identities	$\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$