1) Note that $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ for $n \geq 1$. Therefore $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ equals

(A) 0 **(B)** 1 **(C)** $\frac{3}{2}$ **(D)** $\frac{3}{4}$ **(E)** ∞

2) Which of the following statements about series is true?

(A) If $\lim_{n\to\infty}(u_n) = 0$, then $\sum u_n$ converges.
(B) If $\lim_{n\to\infty}(u_n) \neq 0$, then $\sum u_n$ diverges.
(C) If $\sum u_n$ diverges, then $\lim_{n\to\infty}(u_n) \neq 0$
(D) $\sum u_n$ converges if and only if $\lim_{n\to\infty}(u_n) = 0$
(E) none of these

3) Which of the following series diverges?

(A) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ **(B)** $\sum_{n=1}^{\infty} \frac{n+1}{n!}$ **(C)** $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

(D) $\sum_{n=1}^{\infty} \frac{\ln(n)}{2^n}$ **(E)** $\sum_{n=1}^{\infty} \frac{n}{2^n}$

4) Which of the following series diverges?

(A) $\sum \frac{1}{n^2}$ **(B)** $\sum \frac{1}{n^2+n}$ **(C)** $\sum \frac{n}{n^3+1}$ **(D)** $\sum \frac{n}{\sqrt{4n^2-1}}$ **(E)** none of these

5) Which of the following alternating series diverges?

(A) $\sum \frac{(-1)^{n-1}}{n}$ **(B)** $\sum \frac{(-1)^{n+1}(n-1)}{n+1}$ **(C)** $\sum \frac{(-1)^{n+1}}{\ln(n+1)}$ **(D)** $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$ **(E)** $\sum \frac{(-1)^{n-1}(n)}{n^2+1}$

6) Which of the following statements is true?

(A) If $\sum u_n$ converges, then so does the series $\sum |u_n|$
(B) If a series is truncated after the nth term, then the error is less than the first term omitted
(C) If the terms of an alternating series decrease, then the series converges
(D) If r <1, then the series $\sum r^n$ converges
(E) none of these

7) The series $\sum_{n=0}^{\infty} n!(x-3)^n$ converges if and only if

(A) x = 0 **(B)** 2 < x < 4 **(C)** x = 3 **(D)** $2 \le x \le 4$ **(E)** x < 2 or x > 4

8) The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

(A) $\frac{-1}{24}$ **(B)** $\frac{1}{24}$ **(C)** $\frac{1}{96}$ **(D)** $\frac{-1}{384}$ **(E)** $\frac{1}{384}$

9) The Taylor polynomial for order 3 at x = 1 for e^x is:

(A) $1+(x-1)+\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}$

(B) $e\left[1+(x-1)+\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}\right]$

(C) $e\left[1+(x+1)+\frac{(x+1)^2}{2!}+\frac{(x+1)^3}{3!}\right]$

(D) $e\left[1+(x-1)+\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}\right]$

(E) $e\left[1-(x-1)+\frac{(x-1)^2}{2!}+\frac{(x-1)^3}{3!}\right]$

10) Which of the following series can be used to compute ln (0.8) ?

(A) ln (x-1) expanded about x=0
(B) ln (x) about x = 0
(C) ln (x) in powers of (x-1)
(D) ln (x-1) in powers of (x-1)
(E) none of these

11) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{2^n} \cdot \frac{n^n}{n!}$ is:

(A) 0 **(B)** 2 **(C)** $\frac{2}{e}$ **(D)** $\frac{e}{2}$ **(E)** ∞

12) If the approximate formula $\sin(x) = x - \frac{x^3}{3!}$ is used and | x | < 1 (radian), then the error is numerically less than:

(A) 0.001 **(B)** 0.003 **(C)** 0.005 **(D)** 0.008 **(E)** 0.009

13) If a suitable series is used, then $\int_0^{0.2} \frac{e^{-x}-1}{x}dx$, correct to three decimal places is

(A) -0.200 **(B)** 0.180 **(C)** 0.190 **(D)** -0.190 **(E)** -0.990

14) Given the function $f(x)=\sum_{n=0}^{\infty} a_n x^n$ and $f'(x)=-f(x)$ for all x. If $f(0)=1$ then $f(0.2)$ (correct to three decimal places) is:

(A) 0.905 **(B)** 1.221 **(C)** 0.819 **(D)** 0.820 **(E)** 1.220

15) The sum of the series $\sum_{n=1}^{\infty}\left(\frac{\pi^3}{3^\pi}\right)^n$ is equal to

(A) 0 **(B)** 1 **(C)** $\frac{3^\pi}{\pi^3-3^\pi}$ **(D)** $\frac{\pi^3}{3^\pi-\pi^3}$ **(E)** none of these

16) When $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{3n-1}$ is approximated by the sum of its first 300 terms, the error is closest to:

(A) 0.001 **(B)** 0.002 **(C)** 0.005 **(D)** 0.010 **(E)** 0.020

17) The third order Taylor polynomial $P_3(x)$ for sin (x) about $\frac{\pi}{4}$ is:

(A) $\frac{1}{\sqrt{2}}\left[(x-\frac{\pi}{4})-\frac{1}{3!}(x-\frac{\pi}{4})^3\right]$

(B) $\frac{1}{\sqrt{2}}\left[1+(x-\frac{\pi}{4})-\frac{1}{2}(x-\frac{\pi}{4})^2+\frac{1}{3!}(x-\frac{\pi}{4})^3\right]$

(C) $\frac{1}{\sqrt{2}}\left[1+(x-\frac{\pi}{4})-\frac{1}{2!}(x-\frac{\pi}{4})^2-\frac{1}{3!}(x-\frac{\pi}{4})^3\right]$

(D) $\left[1+(x-\frac{\pi}{4})-\frac{1}{2}(x-\frac{\pi}{4})^2-\frac{1}{6}(x-\frac{\pi}{4})^3\right]$

(E) $\frac{1}{\sqrt{2}}\left(1+x-\frac{x^2}{2!}-\frac{x^3}{3!}\right)$

18) If the series $\tan^{-1}(1)=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots$ is used to approximate $\frac{\pi}{4}$ with an error less than 0.001, then the smallest number of terms needed is

(A) 100 **(B)** 200 **(C)** 300 **(D)** 400 **(E)** 500