Name: _____ Date: _____

Review, through Section 9.2

These questions represent a review of integration techniques, applications of integration, use of l'Hopital's rule, and the beginning of infinite series. If there are any errors, please bring them to my attention. All problems should be done **without** a calculator.

1. The area of the region in the first quadrant enclosed by the graph of $y = x(1-x)$ and the x-axis is

bc-88-1

A) $\frac{1}{6}$

B) $\frac{1}{3}$

C) $\frac{2}{3}$

D) $\frac{5}{6}$

E) 1

2. $\int_0^1 x\left(x^2+2\right)^2\,dx =$

bc-88-2

A) $\frac{19}{2}$

B) $\frac{19}{3}$

C) $\frac{9}{2}$

D) $\frac{19}{6}$

E) $\frac{1}{6}$

3. If $f(x) = \ln\left(\sqrt{x}\right)$, then $f''(x) =$

bc-88-3

A) $-\frac{2}{x^2}$

B) $-\frac{1}{2x^2}$

C) $-\frac{1}{2x}$

D) $-\frac{1}{2x^{\frac{3}{2}}}$

E) $\frac{2}{x^2}$

4. If u, v, and w are nonzero differentiable functions, then the derivative of $\frac{uv}{w}$ is

88-bc-4

A) $\frac{uv' + u'v}{w'}$

B) $\frac{u'v'w + uvw'}{w^2}$

C) $\frac{uvw' - uv'w - u'vw}{w^2}$

D) $\frac{u'vw + uv'w + uvw'}{w^2}$

E) $\frac{uv'w + u'vw - uvw'}{w^2}$

5. $\int_2^\infty \frac{dx}{x^2}$ is

bc-88-7

A) $\frac{1}{2}$

B) $\ln 2$

C) 1

D) 2

E) nonexistent

6. A particle travels in a straight line with a constant acceleration of 3 meters per second per second. If the velocity of the particle is 10 meters per second at time 2 seconds, how far does the particle travel during the time interval when its velocity increases from 4 meters per second to 10 meters per second?

bc-88-12

A) 20 m
B) 14 m
C) 7 m
D) 6 m
E) 3 m

7. $\sin(2x) =$

bc-88-13

A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!} + \ldots$

B) $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \ldots + \frac{(-1)^{n-1} (2x)^{2n-1}}{(2n-1)!} + \ldots$

C) $-\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + \frac{(-1)^n (2x)^{2n}}{(2n)!} + \ldots$

D) $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

E) $2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots + \frac{(2x)^{2n-1}}{(2n-1)!} + \ldots$

8. If $F(x) = \int_1^{x^2} \sqrt{1+t^3}\, dt$, then $F'(x) =$

bc-88-14

A) $2x\sqrt{1+x^6}$
B) $2x\sqrt{1+x^3}$
C) $\sqrt{1+x^6}$
D) $\sqrt{1+x^3}$
E) $\int_1^{x^2} \frac{3t^2}{2\sqrt{1+t^3}}\, dt$

9. $\int xe^{2x}dx =$

bc-88-16

A) $\frac{xe^{2x}}{2}-\frac{e^{2x}}{4}+C$

B) $\frac{xe^{2x}}{2}-\frac{e^{2x}}{2}+C$

C) $\frac{xe^{2x}}{2}+\frac{e^{2x}}{4}+C$

D) $\frac{xe^{2x}}{2}+\frac{e^{2x}}{2}+C$

E) $\frac{x^2e^{2x}}{4}+C$

10. $\int_2^3 \frac{3}{(x-1)(x+2)}dx =$

bc-88-17

A) $-\frac{33}{20}$

B) $-\frac{9}{20}$

C) $\ln\left(\frac{5}{2}\right)$

D) $\ln\left(\frac{8}{5}\right)$

E) $\ln\left(\frac{2}{5}\right)$

11. If three equal subdivisions of $[-4, 2]$ are used, what is the trapezoidal approximation of $\int_{-4}^{2} \frac{e^{-x}}{2}\,dx$?

bc-88-18

A) $e^2 + e^0 + e^{-2}$
B) $e^4 + e^2 + e^0$
C) $e^4 + 2e^2 + 2e^0 + e^{-2}$
D) $\frac{1}{2}\left(e^4 + e^2 + e^0 + e^{-2}\right)$
E) $\frac{1}{2}\left(e^4 + 2e^2 + 2e^0 + e^{-2}\right)$

12. The average value of $\frac{1}{x}$ on the closed interval $[1, 3]$ is

A) $\frac{1}{2}$
B) $\frac{2}{3}$
C) $\frac{\ln 2}{2}$
D) $\frac{\ln 3}{2}$
E) $\ln 3$

13. If c is the number that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - 2x^2$ on the interval $0 \le x \le 2$, then $c =$

88-bc-24

A) 0
B) $\frac{1}{2}$
C) 1
D) $\frac{4}{3}$
E) 2

14. The base of a solid is the region in the first quadrant enclosed by the parabola $y = 4x^2$, the line $x = 1$, and the *x*-axis. Each plane section of the solid perpendicular to the x-axis is a square. The volume of the solid is

bc-88-25

A) $\frac{4\pi}{3}$

B) $\frac{16\pi}{5}$

C) $\frac{4}{3}$

D) $\frac{16}{5}$

E) $\frac{64}{5}$

15. If f is a function such that $f'(x)$ exists for all x and $f(x) > 0$ for all x, which of the following is NOT necessarily true?

bc-88-26

A) $\int_{-1}^{1} f(x)\,dx > 0$

B) $\int_{-1}^{1} 2f(x)\,dx = 2\int_{-1}^{1} f(x)\,dx$

C) $\int_{-1}^{1} f(x)\,dx = 2\int_{0}^{1} f(x)\,dx$

D) $\int_{-1}^{1} f(x)\,dx = -\int_{1}^{-1} f(x)\,dx$

E) $\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$

16. The region *R* in the first quadrant is enclosed by the lines $x = 0$ and $y = 5$ and the graph of $y = x^2 + 1$. The volume of the solid generated when *R* is revolved about the y-axis is

bc-88-29

A) 6π

B) 8π

C) $\frac{34\pi}{3}$

D) 16π

E) $\frac{544\pi}{15}$

17. $\sum_{i=n}^{\infty}\left(\frac{1}{3}\right)^{i} =$

bc-88-30

A) $\frac{3}{2}-\left(\frac{1}{3}\right)^{n}$

B) $\frac{3}{2}\left[1-\left(\frac{1}{3}\right)^{n}\right]$

C) $\frac{3}{2}\left(\frac{1}{3}\right)^{n}$

D) $\frac{2}{3}\left(\frac{1}{3}\right)^{n}$

E) $\frac{2}{3}\left(\frac{1}{3}\right)^{n+1}$

18. The length of the curve $y = x^3$ from $x = 0$ to $x = 2$ is given by

bc-88-33

A) $\int_0^2 \sqrt{1+x^6}\,dx$

B) $\int_0^2 \sqrt{1+3x^2}\,dx$

C) $\pi\int_0^2 \sqrt{1+9x^4}\,dx$

D) $2\pi\int_0^2 \sqrt{1+9x^4}\,dx$

E) $\int_0^2 \sqrt{1+9x^4}\,dx$

19. If k is a positive integer, then $\lim_{x\to\infty}\frac{x^k}{e^x}$

bc-88-35

A) 0
B) 1
C) e
D) $k!$
E) nonexistent

20. Let R be the region between the graphs of $y = 1$ and $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{2}$. The volume of the solid obtained by revolving R about the x-axis is given by

bc-88-36

A) $2\pi \int_0^{\frac{\pi}{2}} x \sin x \, dx$

B) $2\pi \int_0^{\frac{\pi}{2}} x \cos x \, dx$

C) $\pi \int_0^{\frac{\pi}{2}} (1 - \sin x)^2 \, dx$

D) $\pi \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

E) $\pi \int_0^{\frac{\pi}{2}} (1 - \sin^2 x) \, dx$

21. The area of the region between the graph of $y = 4x^3 + 2$ and the x-axis from $x = 1$ to $x = 2$ is

bc-85-1

A) 36
B) 23
C) 20
D) 17
E) 9

22. $\int_1^2 \frac{x+1}{x^2+2x} dx =$

A) $\ln 8 - \ln 3$

B) $\frac{\ln 8 - \ln 3}{2}$

C) $\ln 8$

D) $\frac{3 \ln 2}{2}$

E) $\frac{3 \ln 2 + 2}{2}$

23. $\int \sin(2x+3)\,dx =$

bc-85-18

A) $-2\cos(2x+3)+C$

B) $-\cos(2x+3)+C$

C) $-\frac{1}{2}\cos(2x+3)+C$

D) $\frac{1}{2}\cos(2x+3)+C$

E) $\cos(2x+3)+C$

24. If $\int f(x)\sin x\,dx = -f(x)\cos x + \int 3x^2\cos x\,dx$, then $f(x)$ could be

bc-85-21

A) $3x^2$

B) x^3

C) $-x^3$

D) $\sin x$

E) $\cos x$

25. $\lim_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^5+8}dx}{h}$ is

bc-85-23

A) 0

B) 1

C) 3

D) $2\sqrt{2}$

E) nonexistent

26. $\lim_{x\to\frac{\pi}{4}} \dfrac{\sin\left(x-\dfrac{\pi}{4}\right)}{x-\dfrac{\pi}{4}}$ is

bc-85-29

A) 0
B) $\dfrac{1}{\sqrt{2}}$
C) $\dfrac{\pi}{4}$
D) 1
E) nonexistent

27. $\int_{-1}^{1} \dfrac{3}{x^2}\,dx$ is

bc-85-36

A) -6
B) -3
C) 0
D) 6
E) nonexistent

28. The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the *x*-axis are squares, then its volume is

bc-85-39

A) $\dfrac{\left(1-e^{-6}\right)}{2}$
B) $\dfrac{1}{2}e^{-6}$
C) e^{-6}
D) e^{-3}
E) $1-e^{-3}$

29. If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

bc-85-40

A) $\int_1^2 \frac{1-u^2}{u}\,du$

B) $\int_2^4 \frac{1-u^2}{u}\,du$

C) $\int_1^2 \frac{1-u^2}{2u}\,du$

D) $\int_1^2 \frac{1-u^2}{4u}\,du$

E) $\int_2^4 \frac{1-u^2}{2u}\,du$

30. What is the length of the arc of $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$?

bc-85-41

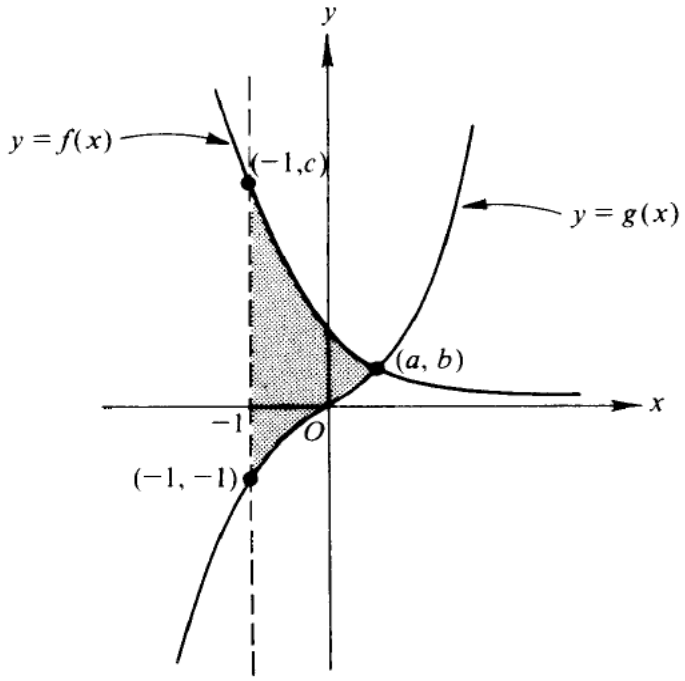
A) $\frac{8}{3}$

B) 4

C) $\frac{14}{3}$

D) $\frac{16}{3}$

E) 7



31. The curves $y = f(x)$ and $y = g(x)$ shown in the figure above intersect at the point (a,b). The area of the shaded region enclosed by these curves and the line $x = -1$ is given by

bc-85-5

A) $\int_0^a \left(f(x) - g(x)\right)dx + \int_{-1}^0 \left(f(x) + g(x)\right)dx$

B) $\int_{-1}^b g(x)\,dx + \int_b^c f(x)\,dx$

C) $\int_{-1}^c \left(f(x) - g(x)\right)dx$

D) $\int_{-1}^a \left(f(x) - g(x)\right)dx$

E) $\int_{-1}^a \left(|f(x)| - |g(x)|\right)dx$

32. For $-1 < x < 1$ if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$, then $f'(x) =$

bc-85-10

A) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n-2}$

B) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n-2}$

C) $\sum_{n=1}^{\infty} (-1)^{2n} x^{2n}$

D) $\sum_{n=1}^{\infty} (-1)^{n} x^{2n}$

E) $\sum_{n=1}^{\infty} (-1)^{n+1} x^{2n}$

33. $\int \frac{dx}{(x-1)(x+2)} =$

bc-85-12

A) $\frac{1}{3} \ln \left| \frac{x-1}{x+2} \right| + C$

B) $\frac{1}{3} \ln \left| \frac{x+2}{x-1} \right| + C$

C) $\frac{1}{3} \ln \left| (x-1)(x+2) \right| + C$

D) $\left(\ln |x-1| \right) \left(\ln |x+2| \right) + C$

E) $\ln \left| (x-1)(x+2)^2 \right| + C$

Answer Key

1. A
2. D
3. B
4. E
5. A
6. B
7. B
8. A
9. A
10. D
11. E
12. D
13. D
14. D
15. C
16. B
17. C
18. E
19. A
20. E
21. D
22. B
23. C
24. B
25. C
26. D
27. E
28. A
29. A
30. C
31. D
32. A
33. A