45. **(a)** None, since $\sin^{-1} x$ is undefined for $x > 1$.

(b) None, since $\sin^{-1} x$ is undefined for $x < -1$.

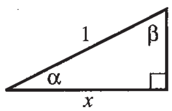
(c) None, since $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \neq 0$.

46. **(a)** None, since $\cos^{-1} x$ is undefined for $x > 1$.

(b) None, since $\cos^{-1} x$ is undefined for $x < -1$.

(c) None, since $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}} \neq 0$.

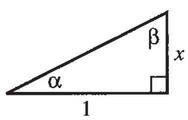
47. **(a)**



$\alpha = \cos^{-1} x, \beta = \sin^{-1} x$

Therefore, $\cos^{-1} x + \sin^{-1} x = \alpha + \beta = \frac{\pi}{2}$.

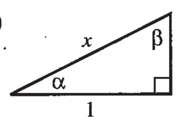
(b)



$\alpha = \tan^{-1} x, \beta = \cot^{-1} x$

Therefore, $\tan^{-1} x + \cot^{-1} x = \alpha + \beta = \frac{\pi}{2}$.

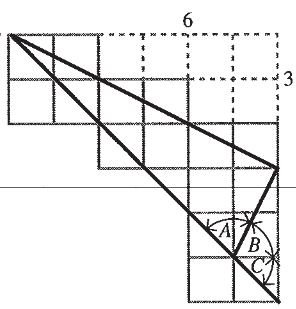
(c)



$\alpha = \sec^{-1} x, \beta = \csc^{-1} x$

Therefore, $\sec^{-1} x + \csc^{-1} x = \alpha + \beta = \frac{\pi}{2}$.

48.



The "straight angle" with the arrows in it is the sum of the three angles A, B, and C.

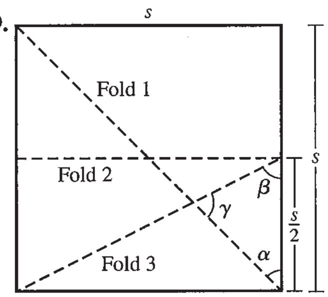
A is equal to $\tan^{-1} 3$ since the opposite side is 3 times as long as the adjacent side.

B is equal to $\tan^{-1} 2$ since the side opposite it is 2 units and the adjacent side is one unit.

C is equal to $\tan^{-1} 1$ since both the opposite and adjacent sides are one unit long.

But the sum of these three angles is the "straight angle," which has measure π radians.

49.



If s is the length of a side of the square, then

$\tan \alpha = \frac{s}{s} = 1$, so $\alpha = \tan^{-1} 1$ and

$\tan \beta = \frac{s}{\frac{s}{2}} = 2$, so $\beta = \tan^{-1} 2$.

Then $\gamma = \pi - \alpha - \beta = \pi - \tan^{-1} 1 - \tan^{-1} 2 = \tan^{-1} 3$.

(In the last step, we used Exercise 48.)

Section 3.9 Derivatives of Exponential and Logarithmic Functions (pp. 172–180)

Exploration 1 Leaving Milk on the Counter

1. The temperature of the refrigerator is 42°F, the temperature of the milk at time $t = 0$.
2. The temperature of the room is 72°F, the limit to which y tends as t increases.
3. The milk is warming up the fastest at $t = 0$. The second derivative $y'' = -30(\ln(0.98))^2(0.98)^t$ is negative, so y' (the rate at which the milk is warming) is maximized at the lowest value of t.
4. We set $y = 55$ and solve;

$$72 - 30(0.98)^t = 55$$

$$(0.98)^t = \frac{17}{30}$$

$$t\ln(0.98) = \ln\left(\frac{17}{30}\right)$$

$$t = \frac{\ln\left(\frac{17}{30}\right)}{\ln(0.98)} = 28.114$$

The milk reaches a temperature of 55° F after about 28 minutes.

5. $\frac{dy}{dt} = -30\ln(0.98) \cdot (0.98)^t$. At $t = 28.114$,

$\frac{dy}{dt} \approx 0.343$ degrees/minute.

Quick Review 3.9

1. $\log_5 8 = \frac{\ln 8}{\ln 5}$
2. $7^x = e^{\ln 7^x} = e^{x \ln 7}$

3. $\ln(e^{\tan x}) = \tan x$

4. $\ln(x^2-4) - \ln(x+2) = \ln\frac{x^2-4}{x+2}$
$= \ln\frac{(x+2)(x-2)}{x+2} = \ln(x-2)$

5. $\log_2(8^{x-5}) = \log_2(2^3)^{x-5} = \log_2 2^{3x-15} = 3x-15$

6. $\frac{\log_4 x^{15}}{\log_4 x^{12}} = \frac{15\log_4 x}{12\log_4 x} = \frac{15}{12} = \frac{5}{4},\ x>0$

7. $3\ln x - \ln 3x + \ln(12x^2) = \ln x^3 - \ln 3x + \ln(12x^2)$
$= \ln\frac{(x^3)(12x^2)}{3x} = \ln(4x^4)$

8. $3^x = 19$
$\ln 3^x = \ln 19$
$x\ln 3 = \ln 19$
$x = \frac{\ln 19}{\ln 3} \approx 2.68$

9. $5^t \ln 5 = 18$
$5^t = \frac{18}{\ln 5}$
$\ln 5^t = \ln\frac{18}{\ln 5}$
$t\ln 5 = \ln 18 - \ln(\ln 5)$
$t = \frac{\ln 18 - \ln(\ln 5)}{\ln 5} \approx 1.50$

10. $3^{x+1} = 2x$
$\ln 3^{x+1} = \ln 2^x$
$(x+1)\ln 3 = x\ln 2$
$x(\ln 3 - \ln 2) = -\ln 3$
$x = \frac{\ln 3}{\ln 2 - \ln 3} \approx -2.71$

Section 3.9 Exercises

1. $\frac{dy}{dx} = \frac{d}{dx}(2e^x) = 2e^x$

2. $\frac{dy}{dx} = \frac{d}{dx}(e^{2x}) = e^{2x}\frac{d}{dx}(2x) = 2e^{2x}$

3. $\frac{dy}{dx} = \frac{d}{dx}e^{-x} = e^{-x}\frac{d}{dx}(-x) = -e^{-x}$

4. $\frac{dy}{dx} = \frac{d}{dx}e^{-5x} = e^{-5x}\frac{d}{dx}(-5x) = -5e^{-5x}$

5. $\frac{dy}{dx} = \frac{d}{dx}e^{2x/3} = e^{2x/3}\frac{d}{dx}\left(\frac{2x}{3}\right) = \frac{2}{3}e^{2x/3}$

6. $\frac{dy}{dx} = \frac{d}{dx}e^{-x/4} = e^{-x/4}\frac{d}{dx}\left(-\frac{x}{4}\right) = -\frac{1}{4}e^{-x/4}$

7. $\frac{dy}{dx} = \frac{d}{dx}(xe^2) - \frac{d}{dx}(e^x) = e^2 - e^x$

8. $\frac{dy}{dx} = \frac{d}{dx}(x^2e^x) - \frac{d}{dx}(xe^x)$
$= (x^2)(e^x) + (e^x)(2x) - [(x)(e^x) + (e^x)(1)]$
$= x^2e^x + xe^x - e^x$

9. $\frac{dy}{dx} = \frac{d}{dx}e^{\sqrt{x}} = e^{\sqrt{x}}\frac{d}{dx}(\sqrt{x}) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

10. $\frac{dy}{dx} = \frac{d}{dx}e^{(x^2)} = e^{(x^2)}\frac{d}{dx}(x^2) = 2xe^{(x^2)}$

11. $\frac{dy}{dx} = \frac{d}{dx}8^x = 8^x\ln 8$

12. $\frac{dy}{dx} = \frac{d}{dx}9^{-x} = 9^{-x}(\ln 9)\frac{d}{dx}(-x) = -9^{-x}\ln 9$

13. $\frac{dy}{dx} = \frac{d}{dx}3^{\csc x} = 3^{\csc x}(\ln 3)\frac{d}{dx}(\csc x)$
$= 3^{\csc x}(\ln 3)(-\csc x\cot x)$
$= -3^{\csc x}(\ln 3)(\csc x\cot x)$

14. $\frac{dy}{dx} = \frac{d}{dx}3^{\cot x} = 3^{\cot x}(\ln 3)\frac{d}{dx}(\cot x)$
$= 3^{\cot x}(\ln 3)(-\csc^2 x)$
$= -3^{\cot x}(\ln 3)(\csc^2 x)$

15. $\frac{dy}{dx} = \frac{d}{dx}\ln(x^2) = \frac{1}{x^2}\frac{d}{dx}(x^2) = \frac{1}{x^2}(2x) = \frac{2}{x}$

16. $\frac{dy}{dx} = \frac{d}{dx}(\ln x)^2 = 2\ln x\frac{d}{dx}(\ln x) = \frac{2\ln x}{x}$

17. $\frac{dy}{dx} = \frac{d}{dx}\ln(x^{-1}) = \frac{d}{dx}(-\ln x) = -\frac{1}{x},\ x>0$

18. $\frac{dy}{dx} = \frac{d}{dx}\ln\frac{10}{x} = \frac{d}{dx}(\ln 10 - \ln x) = 0 - \frac{1}{x}$
$= -\frac{1}{x},\ x>0$

19. $\frac{d}{dx}\ln(\ln x) = \frac{1}{\ln x}\frac{d}{dx}\ln x = \frac{1}{\ln x}\cdot\frac{1}{x} = \frac{1}{x\ln x}$

20. $\frac{dy}{dx} = \frac{d}{dx}(x\ln x - x) = (x)\left(\frac{1}{x}\right) + (\ln x)(1) - 1$
$= 1 + \ln x - 1 = \ln x$

21. $\frac{dy}{dx} = \frac{d}{dx}(\log_4 x^2) = \frac{d}{dx}\frac{\ln x^2}{\ln 4} = \frac{d}{dx}\left[\left(\frac{2}{\ln 4}\right)(\ln x)\right]$
$= \frac{2}{\ln 4}\cdot\frac{1}{x} = \frac{2}{x\ln 4} = \frac{1}{x\ln 2}$

22. $\frac{dy}{dx} = \frac{d}{dx}(\log_5\sqrt{x}) = \frac{d}{dx}\frac{\ln x^{1/2}}{\ln 5} = \frac{d}{dx}\frac{\frac{1}{2}\ln x}{\ln 5}$
$= \frac{1}{2\ln 5}\frac{d}{dx}(\ln x) = \frac{1}{2\ln 5}\cdot\frac{1}{x} = \frac{1}{2x\ln 5},\ x>0$

23. $\frac{dy}{dx} = \frac{d}{dx}\log_2\left(\frac{1}{x}\right) = \frac{d}{dx}(-\log_2 x) = -\frac{1}{x\ln 2}, x > 0$

24. $\frac{dy}{dx} = \frac{d}{dx}\frac{1}{\log_2 x} = -\frac{1}{(\log_2 x)^2}\frac{d}{dx}(\log_2 x)$

$= -\frac{1}{(\log_2 x)^2}\frac{1}{x\ln 2} = -\frac{1}{x(\ln 2)(\log_2 x)^2}$

or $-\frac{\ln 2}{x(\ln x)^2}$

25. $\frac{dy}{dx} = \frac{d}{dx}(\ln 2 \bullet \log_2 x) = (\ln 2)\frac{d}{dx}(\log_2 x)$

$= (\ln 2)\left(\frac{1}{x\ln 2}\right) = \frac{1}{x}, x > 0$

26. $\frac{dy}{dx} = \frac{d}{dx}\log_3(1 + x\ln 3)$

$= \frac{1}{(1+x\ln 3)\ln 3}\frac{d}{dx}(1 + x\ln 3)$

$= \frac{\ln 3}{(1+x\ln 3)\ln 3} = \frac{1}{1+x\ln 3}, \; x > -\frac{1}{\ln 3}$

27. $\frac{dy}{dx} = \frac{d}{dx}(\log_{10} e^x) = \frac{d}{dx}(x\log_{10} e) = \log_{10} e = \frac{\ln e}{\ln 10}$

$= \frac{1}{\ln 10}$

28. $\frac{dy}{dx} = \frac{d}{dx}\ln 10^x = \frac{d}{dx}(x\ln 10) = \ln 10$

29. $m = 5$

$y = 3^x + 1$

$y' = 3^x \ln 3 = 5$

$x = 1.379$

$y = 3^{1.379} + 1 = 5.551$

$(1.379,\ 5.551)$

30. $m_2 = -\frac{1}{m_1} = \frac{1}{3}$

$\frac{d}{dx}(2e^x - 1) = 2e^x$

$\frac{1}{3} = 2e^x$

$\frac{1}{6} = e^x$

$x = -\ln 6$

$y = 2e^x - 1$

$y = \frac{2}{6} - 1 = -\frac{2}{3}$

$\left(-\ln 6,\ -\frac{2}{3}\right)$ or

$(-1.792,\ -0.667)$

31. Equation of line: $y = mx$

Slope: $m = \frac{d}{dx}\ln(2x) = \frac{1}{2x} \bullet 2 = \frac{1}{x}$

At the point where the tangent line touches the graph, $y = mx$ and $y = \ln(2x)$

$mx = \ln(2x)$

$\frac{1}{x} \bullet x = \ln(2x)$

$1 = \ln(2x)$

$e^1 = 2x$

$x = \frac{e}{2}$

Therefore, $m = \frac{1}{x} = \frac{2}{e}$.

32. Equation of line: $y = mx$

Slope: $m = \frac{d}{dx}\left(\ln\frac{x}{3}\right) = \frac{1}{x/3}\frac{d}{dx}\left(\frac{x}{3}\right) = \frac{3}{x} \bullet \frac{1}{3} = \frac{1}{x}$

At the point where the tangent line touches the graph, $y = mx$ and $y = \ln\left(\frac{x}{3}\right)$

$mx = \ln\left(\frac{x}{3}\right)$

$\frac{1}{x} \bullet x = \ln\left(\frac{x}{3}\right)$

$1 = \ln\left(\frac{x}{3}\right)$

$e^1 = \frac{x}{3}$

$x = 3e$

Therefore, $m = \frac{1}{x} = \frac{1}{3e}$.

33. $\frac{dy}{dx} = \frac{d}{dx}(x^\pi) = \pi x^{\pi - 1}$

34. $\frac{dy}{dx} = \frac{d}{dx}(x^{1+\sqrt{2}}) = (1+\sqrt{2})x^{1+\sqrt{2}-1} = (1+\sqrt{2})x^{\sqrt{2}}$

35. $\frac{dy}{dx} = \frac{d}{dx}x^{-\sqrt{2}} = -\sqrt{2}x^{-\sqrt{2}-1}$

36. $\frac{dy}{dx} = \frac{d}{dx}x^{1-e} = (1-e)x^{1-e-1} = (1-e)x^{-e}$

37. $\frac{d}{dx}\ln(x+2) = \frac{1}{x+2}\frac{d}{dx}(x+2) = \frac{1}{x+2}$

Domain of f: $x + 2 > 0$

$x > -2$

Domain of f': $x \neq -2$ and $x > -2$, so $x > -2$

38. $\frac{d}{dx}\ln(2x+2)=\frac{1}{2x+2}\cdot\frac{d}{dx}(2x+2)$

$=\frac{1}{2(x+1)}\cdot 2$

$=\frac{1}{x+1}$

Domain of f: $2x+2>0$

$2x>-2$

$x>-1$

Domain of f': $x\neq -1$ and $x>-1$, so $x>-1$

39. $\frac{d}{dx}\ln(2-\cos x)=\frac{1}{2-\cos x}\cdot\frac{d}{dx}(2-\cos x)$

$=\frac{\sin x}{2-\cos x}$

Domain of f: $2-\cos x>0$

$-\cos x>-2$

$\cos x<2$ which is true for all x.

Domain of f': $\cos x\neq 2$ which is true for all x. All real numbers.

All real numbers.

40. $\frac{d}{dx}\ln(x^2+1)=\frac{1}{x^2+1}\frac{d}{dx}(x^2+1)=\frac{2x}{x^2+1}$

Since $x^2+1>0$ for all x,

Domain of f = Domain of f' = all real numbers.

41. $\frac{d}{dx}\log_2(3x+1)=\frac{1}{(3x+1)\ln 2}\cdot\frac{d}{dx}(3x+1)$

$=\frac{3}{(3x+1)\ln 2}$

Domain of f: $3x+1>0$

$x>-1/3$

Domain of f': $3x+1\neq 0$ and $x>-\frac{1}{3}$, so $x>-\frac{1}{3}$

42. First, note that $\log_{10}\sqrt{x+1}=\log_{10}(x+1)^{1/2}$

$=\frac{1}{2}\log_{10}(x+1)$

$\frac{d}{dx}\log_{10}\sqrt{x+1}=\frac{d}{dx}\left[\frac{1}{2}\log_{10}(x+1)\right]$

$=\frac{1}{2}\cdot\frac{1}{(x+1)\ln 10}\cdot\frac{d}{dx}(x+1)$

$=\frac{1}{2(x+1)\ln 10}$

Domain of f: $x+1>0$

$x>-1$

Domain of f': $x\neq -1$ and $x>-1$

so, $x>-1$

43.

$y=(\sin x)^x$

$\ln y=\ln(\sin x)^x$

$\ln y=x\ln(\sin x)$

$\frac{d}{dx}\ln y=\frac{d}{dx}[x\ln(\sin x)]$

$\frac{1}{y}\frac{dy}{dx}=(x)\left(\frac{1}{\sin x}\right)(\cos x)+\ln(\sin x)(1)$

$\frac{dy}{dx}=y[x\cot x+\ln(\sin x)]$

$\frac{dy}{dx}=(\sin x)^x[x\cot x+\ln(\sin x)]$

44.

$y=x^{\tan x}$

$\ln y=\ln(x^{\tan x})$

$\ln y=(\tan x)(\ln x)$

$\frac{d}{dx}\ln y=\frac{d}{dx}[(\tan x)(\ln x)]$

$\frac{1}{y}\frac{dy}{dx}=(\tan x)\left(\frac{1}{x}\right)+(\ln x)(\sec^2 x)$

$\frac{dy}{dx}=y\left[\frac{\tan x}{x}+(\ln x)(\sec^2 x)\right]$

$\frac{dy}{dx}=x^{\tan x}\left[\frac{\tan x}{x}+(\ln x)(\sec^2 x)\right]$

45. $y=\sqrt[5]{\frac{(x-3)^4(x^2+1)}{(2x+5)^3}}=\left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)^{1/5}$

$\ln y=\ln\left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)^{1/5}$

$\ln y=\frac{1}{5}\ln\frac{(x-3)^4(x^2+1)}{(2x+5)^3}$

$\ln y=\frac{1}{5}[4\ln(x-3)+\ln(x^2+1)-3\ln(2x+5)]$

$\frac{d}{dx}(\ln y)=\frac{4}{5}\frac{d}{dx}\ln(x-3)$

$+\frac{1}{5}\frac{d}{dx}\ln(x^2+1)-\frac{3}{5}\frac{d}{dx}\ln(2x+5)$

$\frac{1}{y}\frac{dy}{dx}=\frac{4}{5}\frac{1}{x-3}+\frac{1}{5}\frac{1}{x^2+1}(2x)-\frac{3}{5}\frac{1}{2x+5}(2)$

$\frac{dy}{dx}=y\left(\frac{4}{5(x-3)}+\frac{2x}{5(x^2+1)}-\frac{6}{5(2x+5)}\right)$

$\frac{dy}{dx}=\left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)^{1/5}\cdot$

$\left(\frac{4}{5(x-3)}+\frac{2x}{5(x^2+1)}-\frac{6}{5(2x+5)}\right)$

46. $$y=\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}=\frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}}$$
$$\ln y=\ln\frac{x(x^2+1)^{1/2}}{(x+1)^{2/3}}$$
$$\ln y=\ln x+\frac{1}{2}\ln(x^2+1)-\frac{2}{3}\ln(x+1)$$
$$\frac{d}{dx}\ln y=\frac{d}{dx}\ln x+\frac{1}{2}\frac{d}{dx}\ln(x^2+1)-\frac{2}{3}\frac{d}{dx}\ln(x+1)$$
$$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}+\frac{1}{2}\frac{1}{x^2+1}(2x)-\frac{2}{3}\frac{1}{x+1}(1)$$
$$\frac{dy}{dx}=y\left(\frac{1}{x}+\frac{x}{x^2+1}-\frac{2}{3(x+1)}\right)$$
$$\frac{dy}{dx}=\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\left(\frac{1}{x}+\frac{x}{x^2+1}-\frac{2}{3(x+1)}\right)$$

47. $$y=x^{\ln x}$$
$$\ln y=\ln(x^{\ln x})=\ln x\bullet\ln x=(\ln x)^2$$
$$\frac{1}{y}\frac{dy}{dx}=\frac{d}{dx}(\ln x)^2=2\ln x\bullet\frac{d}{dx}(\ln x)$$
$$=2\ln x\bullet\frac{1}{x}=\frac{2\ln x}{x}$$
$$\frac{dy}{dx}=y\bullet\frac{2\ln x}{x}=\frac{2x^{\ln x}\ln x}{x}$$

48. $$y=x^{(1/\ln x)}$$
$$\ln y=\ln x^{(1/\ln x)}=\frac{1}{\ln x}\bullet\ln x=1$$
$$\frac{d}{dx}(\ln y)=\frac{d}{dx}(1)$$
$$\frac{1}{y}\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=0,\ x>0$$

49. $\frac{dy}{dx}=e^x$

The tangent line passes through $(0,0)$ and (a,e^a) for some value of a, and has slope e^a. Thus

$$\frac{e^a-0}{a-0}=e^a$$

$\frac{e^a}{a}=e^a$, so $a=1$. Therefore the line has slope e^1 and passes through $(1,e)$. It has equation $y-e=e(x-1)$, or $y=ex$.

Therefore, $y=e^1(x-1)+e^1$
$$y=ex$$

50. For $y=xe^x$, we have $y'=(x)(e^x)+(e^x)(1)=(x+1)e^x$, so the normal line through the point (a,ae^a) has slope

$m=-\frac{1}{(a+1)e^a}$ and its equation is

$y=-\frac{1}{(a+1)e^a}(x-a)+ae^a$. The desired normal line includes the point $(0, 0)$, so we have:

$$0=-\frac{1}{(a+1)e^a}(0-a)+ae^a$$
$$0=\frac{a}{(a+1)e^a}+ae^a$$
$$0=a\left(\frac{1}{(a+1)e^a}+e^a\right)$$
$$a=0\text{ or }\frac{1}{(a+1)e^a}+e^a=0$$

The equation $\frac{1}{(a+1)e^a}+e^a=0$ has no solution (as can be seen by graphing $y=\frac{1}{(x+1)e^x}+e^x$ on a calculator), so we need to use $a=0$. The equation of the normal line is

$y=\frac{-1}{(0+1)e^0}(x-0)+0e^0$, or $y=-x$.

51. (a) $P(0)=\frac{300}{1+2^{4-0}}\approx 18$

(b) $$P'(t)=300\frac{d}{dt}(1+2^{4-t})^{-1}$$
$$=-300(1+2^{4-t})^{-2}\bullet(\ln 2)2^{4-t}(-1)$$
$$=\frac{300(\ln 2)\,2^{4-t}}{(1+2^{4-t})^2}$$
$$P'(4)=\frac{300(\ln 2)}{4}=52$$

(c) Graph $P'(t)$ on a graphing calculator. Use TRACE or CALC→ MAXIMUM to find that the maximum of $P'(t)$ is at $t=4$. The rumor spreads at its maximum rate after 4 days; at that time the rumor is spreading at a rate of 52 students per day.

52. (a) $P(0)=\frac{200}{1+e^{5-0}}=1$

(b) $$\frac{d}{dt}200((1+e^{5-t})^{-1})$$
$$=200(-1)(1+e^{5-t})^{-2}\frac{d}{dt}(1+e^{5-t})$$
$$=200(-1)(1+e^{5-t})^{-2}(e^{5-t})(-1)$$
$$=\frac{200e^{5-t}}{(1+e^{5-t})^2}$$
$$P'(4)=\frac{200e^{5-4}}{(1+e^{5-4})^2}=39$$

52. Continued

(c) Graph $P'(t)$ on a graphing calculator. Use TRACE or CALC→ MAXIMUM to find that $P'(t)$ has a maximum at $t = 5$.

$$P'(5) = \frac{200e^{5-5}}{(1+e^{5-5})^2} = 50$$

The flu spreads at its maximum rate after 5 days. At that time, the flu is spreading to 50 students per day.

53. $$\frac{dA}{dt} = 20\frac{d}{dt}\left(\frac{1}{2}\right)^{t/140}$$
$$= 20\frac{d}{dt}2^{-t/140}$$
$$= 20(2^{-t/140})(\ln 2)\frac{d}{dt}\left(-\frac{t}{140}\right)$$
$$= 20(2^{-t/140})(\ln 2)\left(-\frac{1}{140}\right)$$
$$= -\frac{(2^{-t/140})(\ln 2)}{7}$$

At $t = 2$ days, we have $\frac{dA}{dt} = -\frac{(2^{-1/70})(\ln 2)}{7} \approx -0.098$ grams/day. This means that the rate of *decay* is the positive rate of approximately 0.098 grams/day.

54. (a) $\frac{d}{dx}\ln(kx) = \frac{1}{kx}\frac{d}{dx}kx = \frac{k}{kx} = \frac{1}{x}$

(b) $\frac{d}{dx}\ln(kx) = \frac{d}{dx}(\ln k + \ln x)$
$$= 0 + \frac{d}{dx}\ln x = \frac{1}{x}$$

55. (a) Since $f'(x) = 2^x \ln 2, f'(0) = 2^0 \ln 2 = \ln 2$.

(b) $f'(0) = \lim_{h\to 0}\frac{f(h) - f(0)}{h} = \lim_{h\to 0}\frac{2^h - 2^0}{h}$
$$= \lim_{h\to 0}\frac{2^h - 1}{h}$$

(c) Since quantities in parts (a) and (b) are equal,

$$\lim_{h\to 0}\frac{2^h - 1}{h} = \ln 2.$$

(d) By following the same procedure as above using $g(x) = 7^x$, we may see that $\lim_{h\to 0}\frac{7^h - 1}{h} = \ln 7$.

56. Recall that a point (a, b) is on the graph of $y = e^x$ if and only if the point (b, a) is on the graph of $y = \ln x$. Since there are points (x, e^x) on the graph of $y = e^x$ with arbitrarily large x-coordinates, there will be points $(x, \ln x)$ on the graph of $y = \ln x$ with arbitrarily large y-coordinates.

57. False. It is $(\ln 2)2^x$.

58. False. It is $2e^{2x}$.

59. B. $P(0) = \frac{150}{1+e^{4-0}} = 3$

60. D. $x + 3 > 0$

$x > -3$

61. A. $y = \log_{10}(2x - 3)$
$$\frac{dy}{dx} = \frac{1}{(\ln 10)(2x-3)}\frac{d}{dx}(2x-3)$$
$$= \frac{2}{(\ln 10)(2x-3)}$$

62. E. $y = 2^{1-x}$
$$y' = 2^{1-x}(\ln 2)(-1)$$
$$y'(2) = -2^{1-2}(\ln 2)$$
$$y'(2) = -\frac{(\ln 2)}{2}$$

63. (a) The graph y_4 is a horizontal line at $y = a$.

(b) The graph of y_3 is always a horizontal line.

a	2	3	4	5
y_3	0.693147	1.098613	1.386295	1.609439
$\ln a$	0.693147	1.098612	1.386294	1.609438

We conclude that the graph of y_3 is a horizontal line at $y = \ln a$.

(c) $\frac{d}{dx}a^x = a^x$ if and only if $y_3 = \frac{y_2}{y_1} = 1$.

So if $y_3 = \ln a$, then $\frac{d}{dx}a^x$ will equal a^x if and only if $\ln a = 1$, or $a = e$.

(d) $y_2 = \frac{d}{dx}a^x = a^x \ln a$. This will equal $y_1 = a^x$ if and only if $\ln a = 1$, or $a = e$.

64. $\frac{d}{dx}\left(-\frac{1}{2}x^2 + k\right) = -x$ and $\frac{d}{dx}(\ln x + c) = \frac{1}{x}$.

Therefore, at any value of x, where the two curves intersect, the two tangent lines will be perpendicular.

65. (a) Since the line passes through the origin and has slope $\frac{1}{e}$, its equation is $y = \frac{x}{e}$.

(b) The graph of $y = \ln x$ lies below the graph of the line $y = \frac{x}{e}$ for all positive $x \neq e$. Therefore, $\ln x < \frac{x}{e}$ for all positive $x \neq e$.

(c) Multiplying by e, $e \ln x < x$ or $\ln x^e < x$.

(d) Exponentiating both sides of $\ln x^e < x$, we have $e^{\ln x^e} < e^x$, or $x^e < e^x$ for all positive $x \neq e$.

(e) Let $x = \pi$ to see that $\pi^e < e^\pi$. Therefore, e^π is bigger.

Quick Quiz Sections 3.7–3.9

1. E. $y = \frac{9}{2x} - \frac{x^2}{2}$

$$\frac{dy}{dx} = -\frac{9}{2x^2} - x$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -\frac{9}{2(1)^2} - 1 = -\frac{11}{2}$$

2. A. $\frac{dy}{dx} = \frac{d}{dx}(\cos(3x-2))^3$

$$= 3(\cos(3x-2))^2(-\sin(3x-2))(3)$$
$$= -9\cos^2(3x-2)\sin(3x-2)$$

3. C. $\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}(2x))$

$$= \frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}(2x)$$
$$= \frac{2}{\sqrt{1-4x^2}}$$

4. (a) Differentiate implicitly:

$$\frac{d}{dx}(xy^2 - x^3y) = \frac{d}{dx}(6)$$

$$1 \cdot y^2 + x \cdot 2y\frac{dy}{dx} - \left(3x^2y + x^3\frac{dy}{dx}\right) = 0$$

$$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) If $x = 1$, then $y^2 - y = 6$, so $y = -2$ or $y = 3$.

at $(1, -2)$, $\frac{dy}{dx} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3} = 2.$

The tangent line is $y + 2 = 2(x - 1)$.

At $(1, 3)$, $\frac{dy}{dx} = \frac{3(1)^2(3) - 3^2}{2(1)(3) - 1^3} = 0.$

The tangent line is $y = 3$.

(c) The tangent line is vertical where $2xy - x^3 = 0$, which implies $x = 0$ or $y = \frac{x^2}{2}$. There is no point on the curve where $x = 0$. If $y = \frac{x^2}{2}$, then $x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6.$

Then the only solution to this equation is $x = \sqrt[5]{-24}$.

Chapter 3 Review Exercises
(pp. 181–184)

1. $\frac{dy}{dx} = \frac{d}{dx}\left(x^5 - \frac{1}{8}x^2 + \frac{1}{4}x\right) = 5x^4 - \frac{1}{4}x + \frac{1}{4}$

2. $\frac{dy}{dx} = \frac{d}{dx}(3 - 7x^3 + 3x^7) = -21x^2 + 21x^6$

3. $\frac{dy}{dx} = \frac{d}{dx}(2\sin x\cos x)$

$$= 2(\sin x)\frac{d}{dx}(\cos x) + 2(\cos x)\frac{d}{dx}(\sin x)$$
$$= -2\sin^2 x + 2\cos^2 x$$

Alternate solution:

$$\frac{dy}{dx} = \frac{d}{dx}(2\sin x\cos x) = \frac{d}{dx}\sin 2x = (\cos 2x)(2)$$
$$= 2\cos 2x$$

4. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x+1}{2x-1}\right) = \frac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2} = -\frac{4}{(2x-1)^2}$

5. $\frac{ds}{dt} = \frac{d}{dt}\cos(1-2t) = -\sin(1-2t)(-2) = 2\sin(1-2t)$

6. $\frac{ds}{dt} = \frac{d}{dt}\cot\left(\frac{2}{t}\right) = -\csc^2\left(\frac{2}{t}\right)\frac{d}{dt}\left(\frac{2}{t}\right) = -\csc^2\left(\frac{2}{t}\right)\left(-\frac{2}{t^2}\right)$

$$= \frac{2}{t^2}\csc^2\left(\frac{2}{t}\right)$$

7. $\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{x} + 1 + \frac{1}{\sqrt{x}}\right) = \frac{d}{dx}(x^{1/2} + 1 + x^{-1/2})$

$$= \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$$

8. $\frac{dy}{dx} = \frac{d}{dx}(x\sqrt{2x+1}) = (x)\left(\frac{1}{2\sqrt{2x+1}}\right)(2) + (\sqrt{2x+1})(1)$

$$= \frac{x + (2x+1)}{\sqrt{2x+1}} = \frac{3x+1}{\sqrt{2x+1}}$$

9. $\frac{dr}{d\theta} = \frac{d}{d\theta}\sec(1+3\theta) = \sec(1+3\theta)\tan(1+3\theta)(3)$

$$= 3\sec(1+3\theta)\tan(1+3\theta)$$

10. $\frac{dr}{d\theta} = \frac{d}{d\theta}\tan^2(3-\theta^2)$

$$= 2\tan(3-\theta^2)\frac{d}{d\theta}\tan(3-\theta^2)$$
$$= 2\tan(3-\theta^2)\sec^2(3-\theta^2)(-2\theta)$$
$$= -4\theta\tan(3-\theta^2)\sec^2(3-\theta^2)$$